

Say we are assigned complex numbers A and B represented by $r_{A} e^{i \alpha}$ and $r_{B} e^{i \beta}$ respectively, and we know them to be also vertices of a certain polygon ABCD suspended in complex Argand space, how then can we discover points C and D if the quantities $r_{1}, r_{2}, \theta_{1}$ and $\theta_{2}$ are also known?

Solving this through the use of vectors is an elegant approach to say the least.
Let $\overrightarrow{O A}=a, \quad \overrightarrow{O B}=b, \quad \overrightarrow{O C}=c$ and $\overrightarrow{O D}=d$
Firstly, compute $\overrightarrow{A B}=\overrightarrow{O B}-\overrightarrow{O A}=b-a=r_{B} e^{i \beta}-r_{A} e^{i \alpha}=\mathrm{Re}^{i \phi}$,
where $|b-a|=R$ and $\arg (b-a)=\phi$

Geometrically speaking, if we multiply the vector $\overrightarrow{A B}$ by a general complex number say $\ell e^{i \delta}$, then the magnitude of this vector will be scaled by $\ell$ units and rotated through $\delta$ radians. (clock-wise or anti-clock-wise depending on whether it is designated positive or negative)

So $\overrightarrow{A C}=\overrightarrow{A B}$ multiplied by $\frac{r_{1}}{R} e^{i \theta_{1}}=\operatorname{Re}^{i \phi}$ multiplied by $\frac{r_{1}}{R} e^{i \theta_{1}}$ and
$\overrightarrow{A D}=\overrightarrow{A B}$ multiplied by $\frac{r_{2}}{R} e^{i \theta_{2}}=\operatorname{Re}^{i \phi}$ multiplied by $\frac{r_{2}}{R} e^{i \theta_{2}}$
As such, we can now attempt to solve for complex numbers C and D by recognizing that
$\overrightarrow{O C}=\overrightarrow{O A}+\overrightarrow{A C}=a+c$ and $\overrightarrow{O D}=\overrightarrow{O A}+\overrightarrow{A D}=a+d$

