

Say we are assigned complex numbers A and B represented by $r_A e^{i\alpha}$ and $r_B e^{i\beta}$ respectively, and we know them to be also vertices of a certain polygon ABCD suspended in complex Argand space, how then can we <u>discover points C and D</u> if the quantities r_1 , r_2 , θ_1 and θ_2 are also known?

Solving this through the use of vectors is an elegant approach to say the least.

Let $\overrightarrow{OA} = a$, $\overrightarrow{OB} = b$, $\overrightarrow{OC} = c$ and $\overrightarrow{OD} = d$ Firstly, compute $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = b - a = r_B e^{i\beta} - r_A e^{i\alpha} = \operatorname{Re}^{i\phi}$, where |b - a| = R and $\arg(b - a) = \phi$

Geometrically speaking, if we multiply the vector \overrightarrow{AB} by a general complex number say $\ell e^{i\delta}$, then the magnitude of this vector will be **scaled** by ℓ units and **rotated** through δ radians. (clock-wise or anti-clock-wise depending on whether it is designated positive or negative)

So
$$\overrightarrow{AC} = \overrightarrow{AB}$$
 multiplied by $\frac{r_1}{R} e^{i\theta_1} = \operatorname{Re}^{i\phi}$ multiplied by $\frac{r_1}{R} e^{i\theta_1}$ and
 $\overrightarrow{AD} = \overrightarrow{AB}$ multiplied by $\frac{r_2}{R} e^{i\theta_2} = \operatorname{Re}^{i\phi}$ multiplied by $\frac{r_2}{R} e^{i\theta_2}$

As such, we can now attempt to solve for complex numbers C and D by recognizing that

$$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} = a + c \text{ and } \overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD} = a + d$$