

How can we prove that a triangle inscribed within a semi-circle is always a right angled one using a vectors method? Here goes:

Firstly, by assigning the centre of the semi-circle to be the origin, we define the general vectors  $\overrightarrow{OA} = a$ ,  $\overrightarrow{OB} = b$ ; in addition, recognise that |a| = |b| since both lengths **represent radii of the** 

## same semi-circle.

Based on the above diagram, we have  $\overrightarrow{AB} = b - a$ ,  $\overrightarrow{AC} = -a - b$ Then  $\overrightarrow{AB} \bullet \overrightarrow{AC} = (b - a) \bullet (-a - b) = -a \bullet b - b \bullet b + a \bullet a + a \bullet b$   $= -b \bullet b + a \bullet a$  $= -|b|^2 + |a|^2 = 0$  ( $\because |a| = |b|$ )

This therefore implies angle  $AOB = \frac{\pi}{2}$  (shown)