

How can we prove that a triangle inscribed within a semi-circle is always a right angled one using a vectors method? Here goes:

Firstly, by assigning the centre of the semi-circle to be the origin, we define the general vectors $\overrightarrow{O A}=a, \overrightarrow{O B}=b$; in addition, recognise that $|a|=|b|$ since both lengths represent radii of the

## same semi-circle.

Based on the above diagram, we have $\overrightarrow{A B}=b-a, \quad \overrightarrow{A C}=-a-b$
Then $\overrightarrow{A B} \bullet \overrightarrow{A C}=(b-a) \bullet(-a-b)=-a \bullet b-b \bullet b+a \bullet a+a \bullet b$

$$
\begin{aligned}
& =-b \bullet b+a \bullet a \\
& =-|b|^{2}+|a|^{2}=0 \quad(\because|a|=|b|)
\end{aligned}
$$

This therefore implies angle $\hat{A O B}=\frac{\pi}{2}$ (shown)

