In this section, we shall examine how to graph $y=\cos ^{2} x$ and $y=\sin ^{2} x$ without the use of the graphic calculator.

$$
y=\cos ^{2} x
$$

By the double angle formula, $\cos 2 x=2 \cos ^{2} x-1 \Rightarrow \cos ^{2} x=\frac{1+\cos 2 x}{2}=\frac{1}{2}+\frac{1}{2} \cos 2 x$
Beginning with the graph of $y=\cos x$, we shall perform transformations in the following order:
(i) Scaling parallel to the $x$-axis by a factor of $\frac{1}{2}$. (Also equivalent to doubling the frequency)
(ii) Scaling parallel to the $y$-axis by a factor of $\frac{1}{2}$.
(iii) Translation parallel to the $y$-axis by $\frac{1}{2}$ unit vertically upwards.

The graph of $y=\cos ^{2} x$ is therefore given below:


$$
y=\sin ^{2} x
$$

Using the identity $\cos ^{2} x+\sin ^{2} x=1 \Rightarrow \sin ^{2} x=1-\cos ^{2} x$, we can perform the following transformations on the graph of $y=\cos ^{2} x$ to obtain the graph of $y=\sin ^{2} x$ :
(i) Reflection in the $x$-axis. (Also equivalent to scaling parallel to the $y$-axis by a factor of -1 .)
(ii) Translation parallel to the $y$-axis by 1 unit vertically upwards.

The graph of $y=\sin ^{2} x$ is therefore given below:


