If one is presented with both the definition of f(x) and that of the composite function fg(x), how exactly can the unknown function g(x) be discovered? This is assuming at the onset that the criterion for the existence of fg(x) is satisfied, ie $R_g \subseteq D_f$. The provision of an actual example best illustrates this.

Eg Let
$$f(x) = 2x-5$$
 and $fg(x) = x^2 - 7x + 11$, then

we can say that $fg(x) = 2g(x) - 5 = x^2 - 7x + 11$

Doing a little shuffling of terms therefore gives $g(x) = \frac{1}{2} [(x^2 - 7x + 11) + 5]$

$$=\frac{1}{2}x^2 - \frac{7}{2}x + 8$$
 (shown)

Unsure if the answer arrived at is accurate? We can always work backwards:

$$fg(x) = f\left(\frac{1}{2}x^2 - \frac{7}{2}x + 8\right) = 2\left(\frac{1}{2}x^2 - \frac{7}{2}x + 8\right) - 5 = x^2 - 7x + 11$$
, so things are correct.

Say, what if a **reverse of sorts** was offered in the question, ie to find the unknown function g(x) if instead f(x) and the composite function gf(x) are given? The solving process isn't all that hard either. Let's consider the above example once again, with a slight modification:

Eg Let
$$f(x) = 2x - 5$$
 and $gf(x) = x^2 - 7x + 11$, then

we can say
$$gf(x) = g(2x-5) = x^2 - 7x + 11$$

In this case, we replace x by $\frac{1}{2}x + \frac{5}{2}$ such that $2x - 5 = 2\left(\frac{1}{2}x + \frac{5}{2}\right) - 5 = x$

Thus, $g(x) = \left(\frac{1}{2}x + \frac{5}{2}\right)^2 - 7\left(\frac{1}{2}x + \frac{5}{2}\right) + 11$

$$=\frac{1}{4}x^2 - x - \frac{1}{4}$$
 (shown)

A quick check to verify the above solution is correct:

$$gf(x) = \frac{1}{4}(2x-5)^2 - (2x-5) - \frac{1}{4} = \frac{1}{4}(4x^2 - 20x + 25) - 2x + 5 - \frac{1}{4} = x^2 - 7x + 11.$$