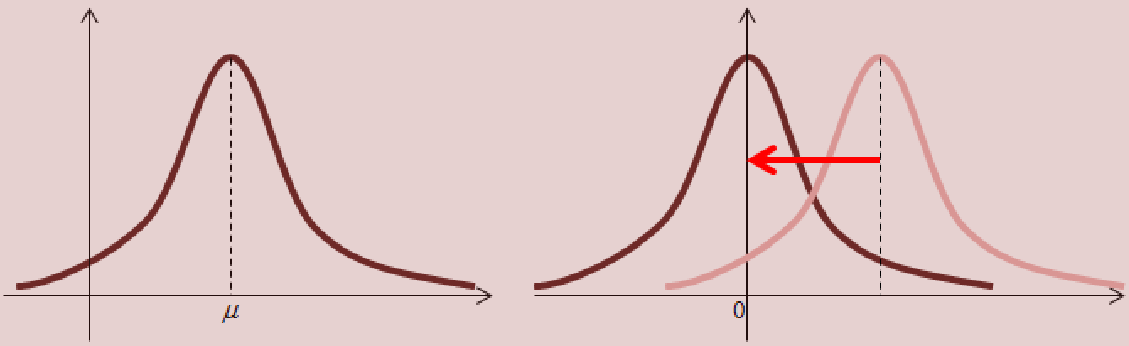
Say a normal random variable X exists such that $X \sim N(\mu, \sigma^2)$ where μ denotes its mean and σ^2 its variance, can the distribution of $X - \mu$ be articulated? Certainly, in fact, it is just simply rendered as $X - \mu \sim N(0, \sigma^2)$, where $X - \mu$ is a normal distribution with its mean centered about the $y-$axis.



Normal curve for random variable X Normal curve for random variable $X - \mu$

If you have problems accepting this, then ask yourself: what exactly does subtracting a constant μ from X really mean? Can you appreciate that it involves nothing more than a shifting of the original normal distribution curve to the left by μ units? In this regard the variance of $X - \mu$ **remains unchanged** at σ^2, and that is because performing a basic translation has no effect whatsoever on the spread of values as far as the curve itself is concerned, ie the curvature (broadness) of the bell shape structure stays the same.

To affirm the validity of this discussion, we can consider a straightforward problem:

For a normal random variable $X \sim N(\mu, \sigma^2)$, compute the probability that X **differs from its mean** by less than a constant a.

Method 1: considering $X \sim N(\mu, \sigma^2)$,

$$P(|X - \mu| < a) = P(-a < X - \mu < a) = P(-a + \mu < X < a + \mu)$$

Standardizing X to $Z \sim N(0, 1)$ therefore gives

$$P\left(\frac{-a+\mu-\mu}{\sigma} < Z < \frac{a+\mu-\mu}{\sigma}\right) = \underline{P\left(\frac{-a}{\sigma} < Z < \frac{a}{\sigma}\right)} \text{-------} (1)$$

Method 2: considering $X-\mu \sim N\left(0,\sigma^2\right)$,

$$P\left(\left|X-\mu\right|<a\right)=P\left(-a<X-\mu<a\right)$$

Standardizing $X-\mu$ to $Z \sim N(0,\,1)$ therefore gives

$$P\left(\frac{-a-0}{\sigma}<Z<\frac{a-0}{\sigma}\right)=\underline{P\left(\frac{-a}{\sigma}<Z<\frac{a}{\sigma}\right)}\text{--------}(2)$$

Can you observe that both (1) and (2) offer the same conclusion?